Experiment 5B: Shear modulus 20.10.2004

1 Introduction

The elastic behavior of beams under stress has long been a fundamental element of both mechanical and civil engineering, and while planar stresses have played a role in many semiconductor devices, the tensile and torsional deformation of solids is now becoming important to semiconductors as well with the rapid development of microsensor devices.

If a homogeneous beam is subjected to a force F applied in the direction of the beams length, the stress in the beam may be calculated as sigma = F/A, where A is the cross sectional area and F is the force in Newtons. If this force causes the beam to lengthen an amount ds, then the strain is defined as

$$\varepsilon = ds/s$$

where s is the original length.
The materials modulus of elasticity (also known as Young's modulus) is defined as

$$E=\sigma/\varepsilon$$

In the elastic region of small deformations

$$F=EAds/s$$

2 Torsional Stress

2.1 Theory

An important case of beam deformation is the case of a beam or bar under torsional stress applied along the axis according to the figure 1a. Experimentally it has been shown that each cross sectional plane moves as a plane. Thus, for example, point B moves to point B1. The interrelated movement of the planes causes a a shear strain τ that increases with the distance from the center, such that $\tau_\rho = 0$ along the axis O-O1 and is a maximum (τ_m) at the outer circumference as shown in Figure 1b and according to the equation

$$\tau_p = \frac{\rho}{r}\tau_m$$

where r is the radius of the beam and ρ is the variable. The shear stress distribution created in the beam may be calculated from the equilibrium condition with the applied moment (the sum of all moments must equal zero). This gives

$$M_v = \int_0^r \rho\tau(\rho)dA,$$

where dA is the differential area ($dA = 2\pi\rho\ d\rho$) and using the earlier equation for the shear stress

$$M_v = \frac{\tau_m}{r}\int_0^r \rho^2 dA = \frac{\tau_m}{r} I_p$$

where I_p is the polar moment of inertia.

Under the application of the torque, the beams free end turns an angle theta relative to the fixed end. Thus, the angle between a fixed plane and a plane turning with the deforming beam is theta, and the amount of deformation varies with the radius as

$$\delta = rd\theta$$

and the shear strain is

$$\gamma = \frac{\delta}{dx} = r\frac{d\theta}{dx}$$

Combining equations for the shear strain and the shear stress gives an equation of the form

$$\gamma = \frac{1}{G}\tau_m$$

where G is the shear modulus. The preceding formulas may be combined

$$\frac{d\theta}{dx} = \frac{M}{GI_p}$$

which can be integrated along the beams length

$$\theta = \frac{M_v L}{GI_p}$$

where L is the length of the beam. This is the formula for the total angular deformation of the beams end under the application of a moment M.

As a final relationship, useful in combination with the results of experiment 5A, the shear modulus and the elastic modulus are related

$$G = \frac{E}{2(1+\upsilon)}$$

where ν is Poission's ratio (~0.3 for most materials). If you have done both experiments, you can use this equation to compare the results.
.

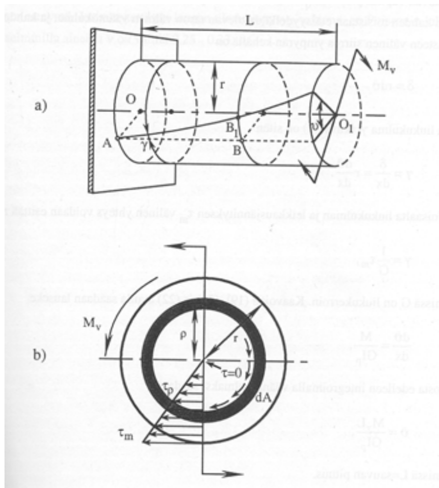

Figure 1 a) A beam subjected to a torsional moment M, b) the cross-section of the beam showing the distribution of the shear strain.

2.2 Experiment 5B: Measurement of the shear modulus

1. Check that the head of the micrometer hits in the middle of the brad in the torque arm.
2. Check that the lamp is lighted as the micrometer hits the brad. If the micrometer indicates less than 25 mm, adjust the position of the rod. (The screw is found from the end of the rod opposite to the torque arm.)
3. Weigh the masses with the digital scale.
4. Measure the diameter of the rod using a caliper. (ask the assistant)
5. Measure the length (l) of the distorting part of the rod.
6. Measure the distance (a) between the point of suspension and the rod.
7. Measure the distance (b) between the micrometer and the rod.
8. Write down the material of the rod.

9. Write down the micrometer indication just as the lamp is lit.
10. Add weights one by one and write down the corresponding indication of the micrometer as well as the mass of the weight. Go on until you have added ten weights.
11. Also measure the distortion as you are removing the weights. Remember to unwind the micrometer before removing the weight.
12. Write down the precision of all measurements.

Instruction for writing the laboratory report:

Calculate the polar moment of inertia
Determine k graphically from the slope of the test results (θ vs L).
Calculate the rigidity modulus and the limits of error by help of (k).
Compare the results with those given in the literature.
Calculate the Poisson ratio and the limits of error using the Young modulus given in the literature.